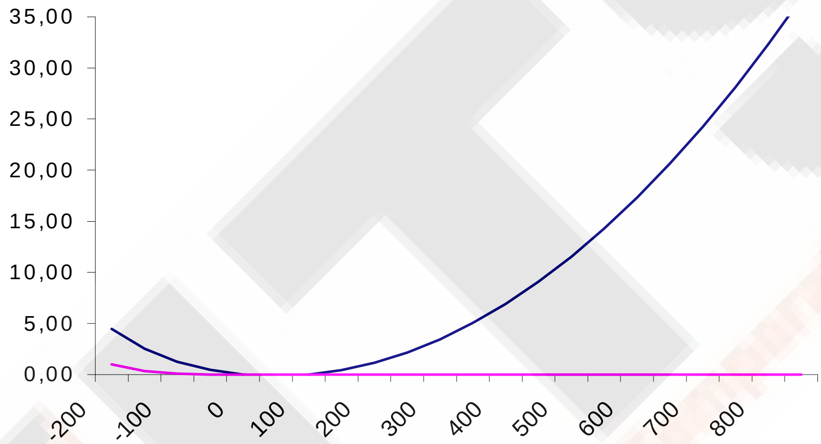
The Callendar – van Dusen coefficients

The platinum thermometer is one of the most linear and practical temperature transducers in existance. Yet it is still necessary to linearise the measured signal, as will appear from the diagram below. The diagram illustrates the disparity in ohms between the actual resistance value at a given temperature and the value that would be obtained by a simple linear calculation for a Pt100 sensor:



Figur 1. Deviation in ohms between the actual resistance value and the linear interpolation as a function of the temperature expressed in °C.

According to IEC751, the non-linearity of the platinum thermometer can be expressed as:

$$R_t = R_0[1 + \mathrm{A}t + \mathrm{B}t^2 + \mathrm{C}(t - 100)t^3] \qquad (1)$$

in which C is only applicable when t < 0 °C.

The coefficients A, B, and C for a standard sensor are stated in IEC751. If a standard sensor is not available or if a greater accuracy is required than can be obtained from the coefficients in the standard, the coefficients can be measured individually for each sensor. This can be done e.g. by determining the resistance value at a number of known temperatures and then determining the coefficients A, B, and C by regression analysis.

The Callendar – van Dusen method:

However, an alternative method for determination of these coefficients exists. This method is based on the measuring of 4 known temperatures:

Measure R_0 at t_0 = 0 °C (the freezing point of water)

Measure R_{100} at t_{100} = 100 °C (the boiling point of water)

Measure R_h at t_h = a high temperature (e.g. the freezing point of zink, 419.53 °C)
Measure R_l at t_l: = a low temperature (e.g. the boiling point of oxygen, -182.96 °C)

Calculation of α:

First the linear parameter α is determined as the normalised slope between 0 and 100 °C:

$$\alpha = \frac{R_{100} - R_0}{100 \cdot R_0} \qquad (2)$$

If this rough approximation is enough, the resistance at other temperatures can be calculated as:

$$R_t = R_0 + R_0 \alpha \cdot t \qquad (3)$$

and the temperature as a function of the resistance value as:

$$t = \frac{R_t - R_0}{R_0 \cdot \alpha} \qquad (4)$$

Calculation of δ:

Callendar has established a better approximation by introducing a term of the second order, δ, into the function. The calculation of δ is based on the disparity between the actual temperature, t_h, and the temperature calculated in (4):

$$\delta = \frac{t_h - \frac{R_{th} - R_0}{R_0 \cdot \alpha}}{(\frac{t_h}{100} - 1)(\frac{t_h}{100})} \qquad (5)$$

With the introduction of δ into the equation, the resistance value for positive temperatures can be calculated with great accuracy:

$$R_t = R_0 + R_0 \alpha \left[t - \delta (\frac{t}{100} - 1)(\frac{t}{100}) \right] \qquad (6)$$

Calculation of β:

At negative temperatures (6) will still give a small deviation as shown in figure 1 (bottom curve). Van Dusen therefore introduced a term of the fourth order, β, which is only applicable for $t < 0$ °C. The calculation of β is based on the disparity between the actual temperature, t_l, and the temperature that would result from employing only α and δ:

$$\beta = \frac{t_l - \left[\frac{R_{tl} - R_0}{R_0 \cdot \alpha} + \delta (\frac{t_l}{100} - 1)(\frac{t_l}{100}) \right]}{(\frac{t_l}{100} - 1)(\frac{t_l}{100})^3} \qquad (7)$$

With the introduction of both Callendar's and van Dusen's constant, the resistance value can be calculated correctly for the entire temperature range, as long as one remembers to set $\beta = 0$ for $t > 0$ °C:

$$R_t = R_0 + R_0 \alpha \left[t - \delta (\frac{t}{100} - 1)(\frac{t}{100}) - \beta (\frac{t}{100} - 1)(\frac{t}{100})^3 \right] \qquad (8)$$

Conversion to A, B, and C:

Equation (8) is the necessary tool for accurate temperature determination. However, seeing that the IEC751 coefficients A, B, and C are more widely used, it would be natural to convert to these coefficients:

Equation (1) can be expanded to:

$$R_t = R_0(1 + At + Bt^2 - 100Ct^3 + Ct^4) \quad (9)$$

and by simple coefficient comparison with equation (8) the following can be determined:

$$A = \alpha + \frac{\alpha \cdot \delta}{100} \quad (10)$$

$$B = -\frac{\alpha \cdot \delta}{100^2} \quad (11)$$

$$C = -\frac{\alpha \cdot \beta}{100^4} \quad (12)$$

As an example, the table below shows both sets of coefficients for a Pt100 resistor according to the IEC751 and ITS90 scale:

α	0,003850	A	$3,908 \times 10^{-3}$
δ	1,4999	B	$-5,775 \times 10^{-7}$
β	0,10863	C	$-4,183 \times 10^{-12}$

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